A TWO-PLY TERMINATION STRATEGY FOR MECHANICALLY COUPLED TAPERED LAMINATES.

Uncoupled, Extension-Shearing and/or Bending-Twisting coupled laminates

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Presentation Contents

- Background, motivation and context.

- A two-ply termination algorithm is employed to develop permissible tapered designs, with:
  - contiguity constraints (≤ 2 adjacent plies with the same orientation)
  - consistent mechanical coupling characteristics
  - immunity to thermal warping distortion

- Tapered results are presented for 3 laminate classes with coupling stiffness matrix, **B** = **0**, i.e.:
  - *Simple* or fully uncoupled, orthotropic laminates,
  - *Bending-Twisting* coupled laminates,
  - *Extension-Shearing*, *Bending-Twisting* coupled laminates.

- Consideration is given to the potential effectiveness of introducing tailored mechanical coupling through ply terminations, e.g.
  - to induce *Bending-Twisting* coupling in parts of a wing box structure, through the introduction of laminate level *Extension-Shearing* coupling, or
  - to introduce laminate level *Bending-Twisting* coupling, where shear buckling strength becomes a design constraint.

- Lamination parameters are introduced to permit an interrogation of the extent of resulting design space for tapered laminates.

- The relative change in compression/shear buckling strength within the tapered laminate design is assessed via a mapping of buckling factor contours onto the lamination parameter design space.

- Conclusions.

Background

The work describe here is part of an on-going study addressing **Mechanically Coupled** (Composite) **Laminates**.

Definitive listings of laminates have been derived for UD material using (but not restricted to) combinations of standard fibre angle orientations, i.e. 0, 90 and/or ±45° (= ±θ°).

The derivation involved the added restrictions that *each layer in the laminate*:

- *has identical material properties;*
- *has identical thickness;*
- *differs only by its orientation.*

Following on from a previous study on tapered laminate designs with single ply terminations[1], two ply terminations are now considered for:

- *Extension-Shearing and/or Bending-Twisting coupled laminates, in which*;
- *the coupling matrix,* $\boldsymbol{B} = \boldsymbol{0}$; and
- *stacking sequence symmetries are unconstrained (but not detailed here).*

[1] York, C. (2015) On tapered warp-free laminates with single-ply terminations. Composites Part A: Applied Science and Manufacturing, 72, pp. 127-138.

Laminate Characterisation

The thermo-mechanical behaviour of coupled laminates may be determined from the specific form of the **extensional** and **bending** stiffness matrices:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{A}_{11} & \mathrm{A}_{12} & \mathrm{A}_{16} \\ & \mathrm{A}_{22} & \mathrm{A}_{26} \\ \text{Sym.} & & \mathrm{A}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{D}_{11} & \mathrm{D}_{12} & \mathrm{D}_{16} \\ & \mathrm{D}_{22} & \mathrm{D}_{26} \\ \text{Sym.} & & \mathrm{D}_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \qquad (1)$$

Couplings exist between:

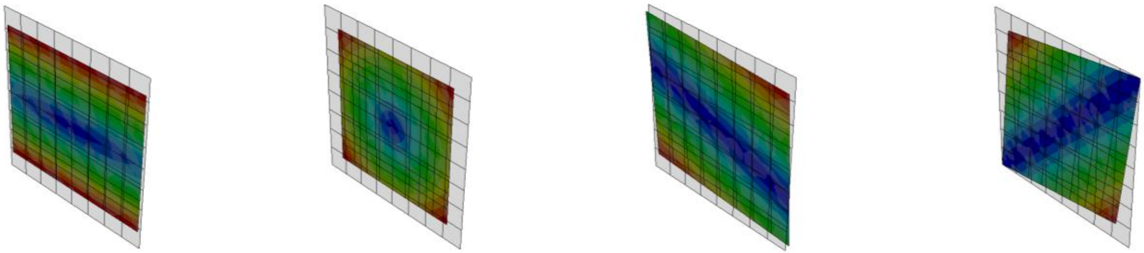
- Extension and Shearing, when A_{16}, $A_{26} \neq 0$, and
- Bending and Twisting, when D_{16}, $D_{26} \neq 0$.

A given laminate can be described in terms of its physical response, due to an applied set of thermal or mechanical force and/or moment resultants …..

Response based labelling for the 3 laminate classes of interest…

Note that laminates with *Extension-Shearing* coupling <u>only</u> were found to possess no tapered solutions within the ply number groupings investigated and are therefore not considered further.

Uncoupled in Extension		***Extension-Shearing***	
Uncoupled in Bending	***Bending-Twisting***	**Uncoupled in Bending**	***Bending-Twisting***



$[+/-_2/\bigcirc/+_2/-]_T$	$[+/-/-/+]_T$	$[\pm/\bigcirc/-/\bigcirc/-_3/\bigcirc/-_3/\bigcirc/+]_T$	$[+/+]_T$
Simple laminate	***<u>B-T</u> coupled laminate***	***<u>E-S</u> coupled laminate***	***<u>E-S</u>;<u>B-T</u> coupled laminate***

Figure 1 – In-plane thermal contraction responses (exaggerated) resulting from a typical high temperature curing process. All examples shown are square, initially flat, composite laminates. The stacking sequences provided, in symbolic form, are representative of the minimum ply number grouping for each laminate class, with standard ply orientations ±45, 0 and 90° in place of symbols +, –, ○ and ●, respectively.

Tapered laminates

Tapered laminate designs have been developed using a two stage algorithm:

The first stage can be described as a top-down process, in which each ply number grouping (n) is algorithmically filtered through ply number groupings with fewer plies, i.e., representing two (n – 2) ply terminations.

> The results from this first stage provide a reduced design space of compatible stacking sequences for *odd* or *even* ply number groupings; *assuming two ply terminations*.

The second stage, which can be described as a bottom-up process, begins with the minimum ply number grouping of interest.

> Stacking sequences from the first stage are now algorithmically filtered through those with higher ply number groupings, in turn, where only compatible sequences with the lower ply number grouping are retained.

The design space of tapered solutions depends on the minimum ply number grouping required in the final tapered laminate design.

> *The second stage must therefore be repeated for each change in the minimum ply number grouping required.*

Examples will help to clarify this….

Table 1 – Results from the two ply termination algorithm applied to ***Simple*** laminates with even ply number groupings.

(1)	(2)	(3)	(4)	(5)	(6)
16	260	↘ 164 (242)	442 (221/-)	**44**	218 (109/-)
14	74	↘ 46 (66)	122 (61/-)	**22**	154 (77/-)
12	22	↘ 10 (18)	24 (12/-)	**10**	24 (12/-)
10	4	↘ 4 (4)	4 (2/-)	**4**	4 (2/-)
8	1	↘ - (1)	-	**1**	-

Column:

(1) Ply number grouping, *n*.

(2) Number of stacking sequences with ply contiguity ≤ 2.

(3) Number of *n* ply laminates from (2) matching *n*-2 ply laminates after top-down termination scheme. Number of *n* ply laminates matching *n*+2 ply laminates are shown in parentheses.

(4) Number of tapered solutions between adjacent ply number groupings (*n*) and orientation (○ or ●/+ & –) of corresponding ply terminations.

(5) Number of *n* ply laminates from (3) matching *n*+2 ply laminates after continuous bottom-up termination scheme. Number of *n* ply laminates matching *n*-2 ply laminates are shown in parentheses.

(6) Number of tapered solutions arising from (5) within each ply number grouping (*n*) and orientation (○ or ●/+ & –) of corresponding ply terminations, for continuous tapering between 8-16 plies.

Table 2 – Results from the two ply termination algorithm applied to ***Simple*** laminates with odd ply number groupings.

(1)	(2)	(3)	(4)	(5)	(6)
15	246	↘ 144 (236)	460 (179/72/30)	**54**	314 (123/56/12)
13	54	↘ 40 (54)	130 (51/16/12)	**28**	118 (47/16/8)
11	18	↘ 14 (18)	38 (15/4/4)	**14**	38 (15/4/4)
9	6	↘ - (6)	-	**6**	-

Column:

(1) Ply number grouping, n.

(2) Number of stacking sequences with ply contiguity ≤ 2.

(3) Number of n ply laminates from (2) matching n-2 ply laminates after top-down termination scheme. Number of n ply laminates matching n+2 ply laminates are shown in parentheses.

(4) Number of tapered solutions between adjacent ply number groupings (n) and orientation (○ or ●/○ & ●/+ & –) of corresponding ply terminations.

(5) Number of n ply laminates from (3) matching n+2 ply laminates after continuous bottom-up termination scheme. Number of n ply laminates matching n-2 ply laminates are shown in parentheses.

(6) Number of tapered solutions arising from (5) within each ply number grouping (n) and orientation (○ or ●/○ & ●/+ & –) of corresponding ply terminations, for continuous tapering between 9-15 plies.

Table 3 – Results from the two ply termination algorithm applied to ***B-T*** coupled laminates with even ply number groupings.

(1)	(2)	(3)	(4)	(5)	(6)
16	5,927	↘ 3,572 (5,927)	11,742 (5,871/-)	**1,002**	7,060 (3,530/-)
14	980	↘ 812 (980)	2,244 (1,122/-)	**354**	1,474 (737/-)
12	203	↘ 142 (203)	426 (213/-)	**116**	386 (193/-)
10	42	↘ 38 (42)	98 (49/-)	**38**	98 (49/-)
8	12	↘ - (12)	-	**12**	-

Column:

(1) Ply number grouping, *n*.

(2) Number of stacking sequences with ply contiguity ≤ 2.

(3) Number of *n* ply laminates from (2) matching *n*-2 ply laminates after top-down termination scheme. Number of *n* ply laminates matching *n*+2 ply laminates are shown in parentheses.

(4) Number of tapered solutions between adjacent ply number groupings (*n*) and orientation (○ or ●/+ & –) of corresponding ply terminations.

(5) Number of *n* ply laminates from (3) matching *n*+2 ply laminates after continuous bottom-up termination scheme. Number of *n* ply laminates matching *n*-2 ply laminates are shown in parentheses.

(6) Number of tapered solutions arising from (5) within each ply number grouping (*n*) and orientation (○ or ●/+ & –) of corresponding ply terminations, for continuous tapering between 8-16 plies.

Table 4 – Results from the two ply termination algorithm applied to ***B-T*** coupled laminates with odd ply number groupings.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
15	6054	↘ 3,462 (6,034)	9,896 (4,071/1,526/228)	954	4,970 (2,007/956/-)	**1,914**	7,644 (3,079/1,346/140)
13	878	↘ 578 (878)	1,564 (615/294/40)	318	1,232 (490/252/-)	**578**	1,564 (615/294/40)
11	148	↘ 96 (148)	272 (102/68/-)	96	272 (102/68/-)	**148**	-
9	28	↘ - (28)	-	28	-	**-**	-

Column:

(1) Ply number grouping, *n*.

(2) Number of stacking sequences with ply contiguity ≤ 2.

(3) Number of *n* ply laminates from (2) matching *n*-2 ply laminates after top-down termination scheme. Number of *n* ply laminates matching *n*+2 ply laminates are shown in parentheses.

(4) Number of tapered solutions between adjacent ply number groupings (*n*) and orientation (○ or ●/○ & ●/+ & –) of corresponding ply terminations.

(5) Number of *n* ply laminates from (3) matching *n*+2 ply laminates after continuous bottom-up termination scheme. Number of *n* ply laminates matching *n*-2 ply laminates are shown in parentheses.

(6) Number of tapered solutions arising from (5) within each ply number grouping (*n*) and orientation (○ or ●/○ & ●/+ & –) of corresponding ply terminations, for continuous tapering between 9-15 plies.

(7)-(8) Repeat of procedure in (5)-(6) for tapering between 11-15 plies.

Table 5 – Results from the two ply termination algorithm applied to ***E-S-B-T*** coupled laminates with even ply number groupings.

(1)	(2)	(3)	(4)	(5)	(6)
16	20,363	↘ 17,305 (20,363)	76,407 (22,034/18,285/14,054)	**8,878**	57,136 (16,237/13,993/10,669)
14	3,551	↘ 2,912 (3,551)	13,451 (3,896/3,257/2,402)	**2,198**	11,923 (3,433/2,955/2,102)
12	675	↘ 611 (675)	2,511 (746/613/406)	**549**	2,416 (711/602/392)
10	149	↘ 137 (149)	458 (138/114/68)	**137**	458 (138/114/68)
8	35	↘ - (35)	-	**35**	-

Column:

(1) Ply number grouping, *n*.

(2) Number of stacking sequences with ply contiguity ≤ 2.

(3) Number of *n* ply laminates from (2) matching *n*-2 ply laminates after top-down termination scheme. Number of *n* ply laminates matching *n*+2 ply laminates are shown in parentheses.

(4) Number of tapered solutions between adjacent ply number groupings (*n*) and orientation (○ or ●/+/–) of corresponding ply terminations.

(5) Number of *n* ply laminates from (3) matching *n*+2 ply laminates after continuous bottom-up termination scheme. Number of *n* ply laminates matching *n*-2 ply laminates are shown in parentheses.

(6) Number of tapered solutions arising from (5) within each ply number grouping (*n*) and orientation (○ or ●/+/–) of corresponding ply terminations, for continuous tapering between 8-16 plies.

Table 6 – Results from the two ply termination algorithm applied to ***E-S-B-T*** coupled laminates with odd ply number groupings.

(1)	(2)	(3)	(4)
13	6307	↘ 5,123 ( )	26,303 (4,919/4,831*/3,861*/1,447/1,029/1,154/1,667)
11	1088	↘ 892 (1088)	4,583 (808/774*/609*/300/208/228/340)
9	197	↘ - (197)	

Column:

(1) Ply number grouping, n.

(2) Number of stacking sequences with ply contiguity ≤ 2.

(3) Number of n ply laminates from (2) matching n-2 ply laminates after top-down termination scheme. Number of n ply laminates matching n+2 ply laminates are shown in parentheses.

(4) Number of tapered solutions between each adjacent ply number grouping (n) and orientation (○ or ●/+/–/○ & + or ● & +/○ & – or ● & –/○ & ●/+ & –) of corresponding ply terminations.

*The numbers of + and – ply terminations differ due to the constraint that the first + (surface) ply is retained throughout the tapering operation.

How might the design engineer handle all this data?....

1. LAMINATION PARAMETER DESIGN SPACE

Lamination parameter design spaces for <u>*B-T*</u> coupled laminates with even ply number groupings, $8 \le n \le 16$, are illustrated in Fig. 1.

These are represented by a 2-dimensional space for the **extensional stiffness**:

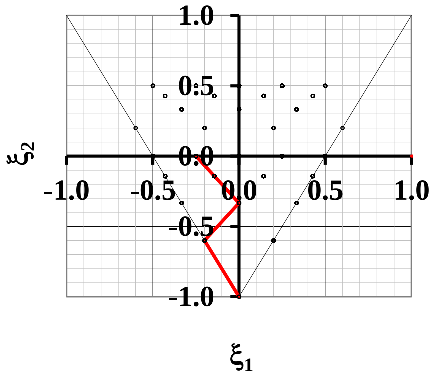


Figure 1(d)

Tapered laminate, highlighted:

+/–/–/●/●/○/●/+/+/●/○/●/●/–/–/+

+/–/–/●/●/○/+/+/○/●/●/–/–/+

+/–/–/●/○/+/+/○/●/–/–/+

+/–/–/●/+/+/●/–/–/+

+/–/–/+/+/–/–/+

$$A_{11} = \{U_1 + \xi_1 U_2 + \xi_2 U_3\} \times (n \times t)$$

$$A_{12} = A_{21} = \{-\xi_2 U_3 + U_4\} \times H$$

$$A_{22} = \{U_1 - \xi_1 U_2 + \xi_2 U_3\} \times H$$

$$A_{66} = \{-\xi_2 U_3 + U_5\} \times H$$

$$U_1 = \{3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}\}/8$$

$$U_2 = \{Q_{11} - Q_{22}\}/2$$

$$U_3 = \{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}\}/8$$

$$U_4 = \{Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}\}/8$$

$$U_5 = \{Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}\}/8$$

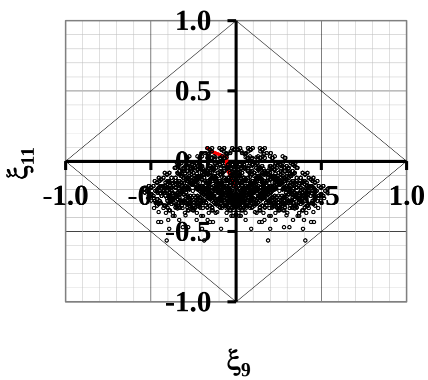
$$Q_{11} = E_1/(1 - \nu_{12}\nu_{21})$$

$$Q_{12} = \nu_{12}E_2/(1 - \nu_{12}\nu_{21})$$

$$Q_{22} = E_2/(1 - \nu_{12}\nu_{21})$$

$$Q_{66} = G_{12}$$

…. and a 3-dimensional space for the **bending stiffness** (illustrated as a three view orthographic projection):



$$D_{11} = \{U_1 + \xi_9 U_2 + \xi_{10} U_3\} \times H^3/12$$

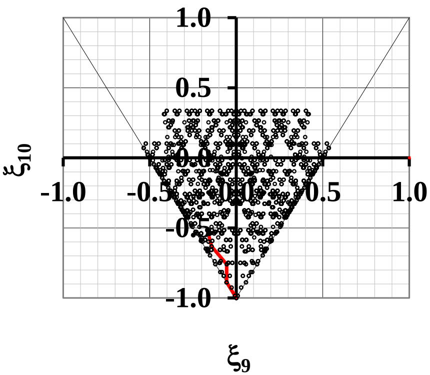
$$D_{12} = \{U_4 - \xi_{10} U_3\} \times H^3/12$$

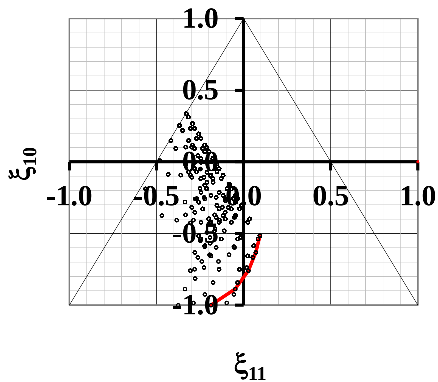
$$D_{16} = \{\xi_{11} U_2/2 + \xi_{12} U_3\} \times H^3/12$$

$$D_{22} = \{U_1 - \xi_9 U_2 + \xi_{10} U_3\} \times H^3/12$$

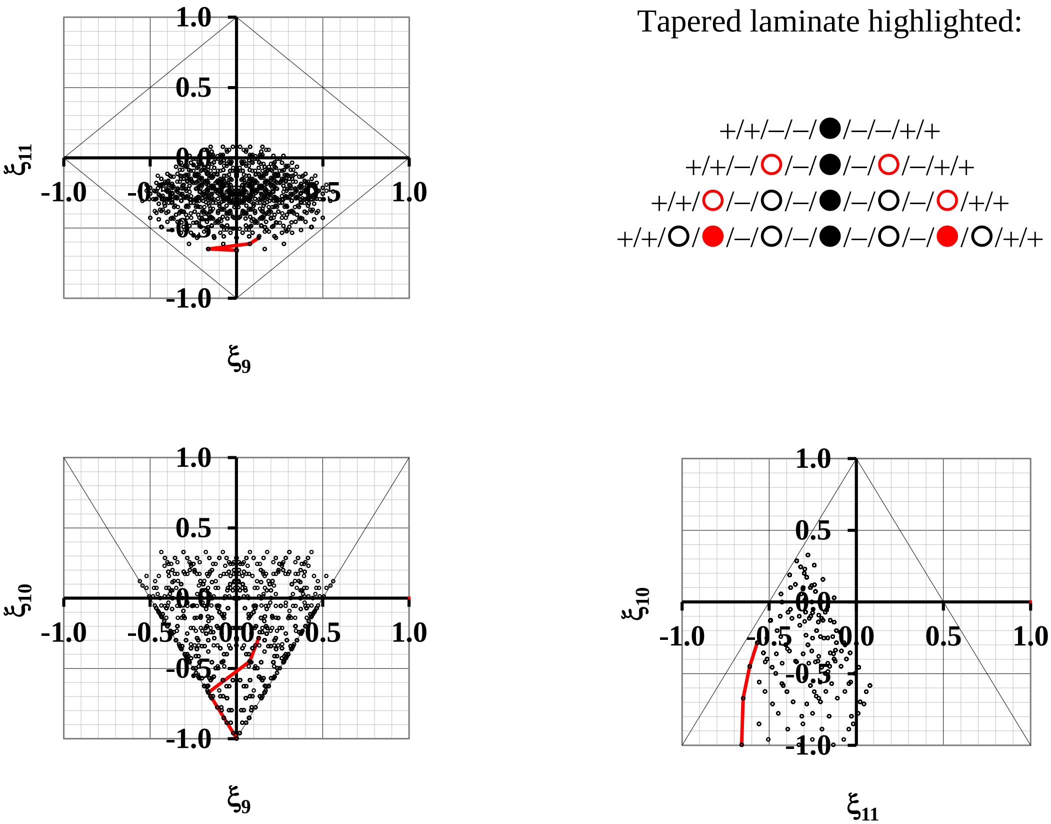
$$D_{26} = \{\xi_{11} U_2/2 - \xi_{12} U_3\} \times H^3/12$$

$$D_{66} = \{-\xi_{10} U_3 + U_5\} \times H^3/12$$

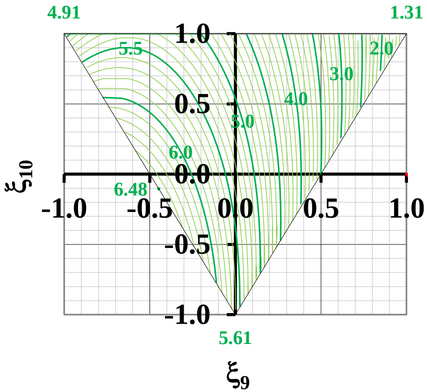




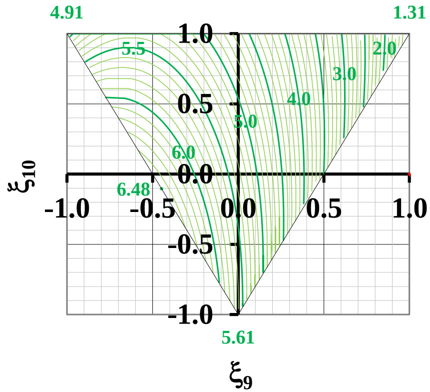
Similarly, odd ply number groupings, $9 \leq n \leq 15$, are illustrated in Fig. 2.



(Addendum) Interpretation of lamination parameter design spaces: cross section at $\xi_{11} = 0$



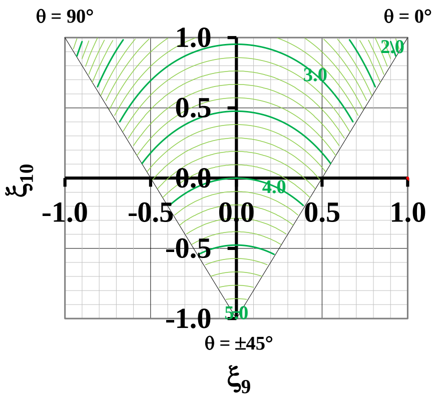
(a) $+k_{xy,\infty}$



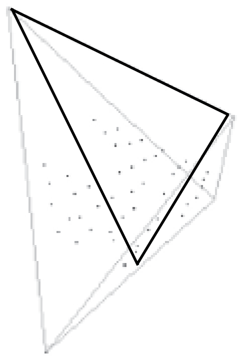
(b) $-k_{xy,\infty}$

Buckling factor mapping for infinitely long simply supported plates.

Compression buckling contours, $k_{x,\infty}$ ($k_{xy,\infty}$) = 4.00 (5.34), for fully isotropic laminates, i.e. $\xi_9 = \xi_{10} = \xi_{11} = 0$

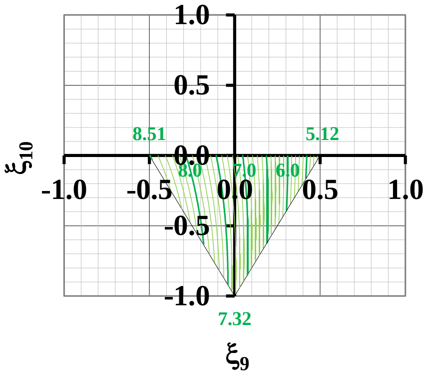


(c) $k_{x,\infty}$

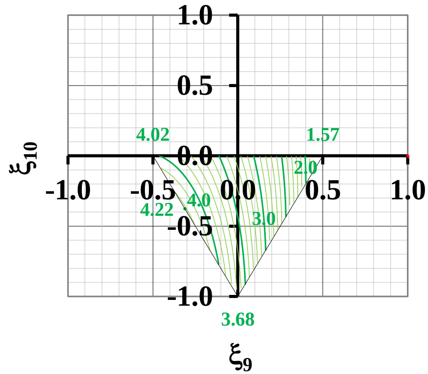


$\xi_{11} = 0$

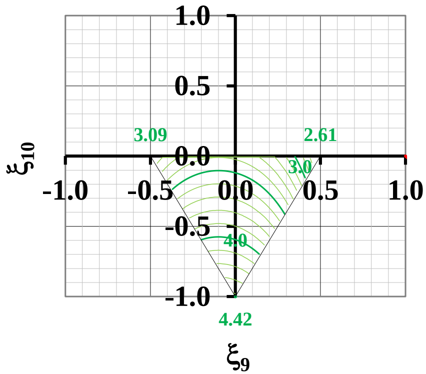
(Addendum) Interpretation of lamination parameter design spaces: cross section at $\xi_{11} = 0.5$



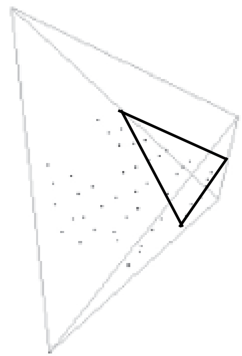
(a) $+k_{xy,\infty}$



(b) $-k_{xy,\infty}$

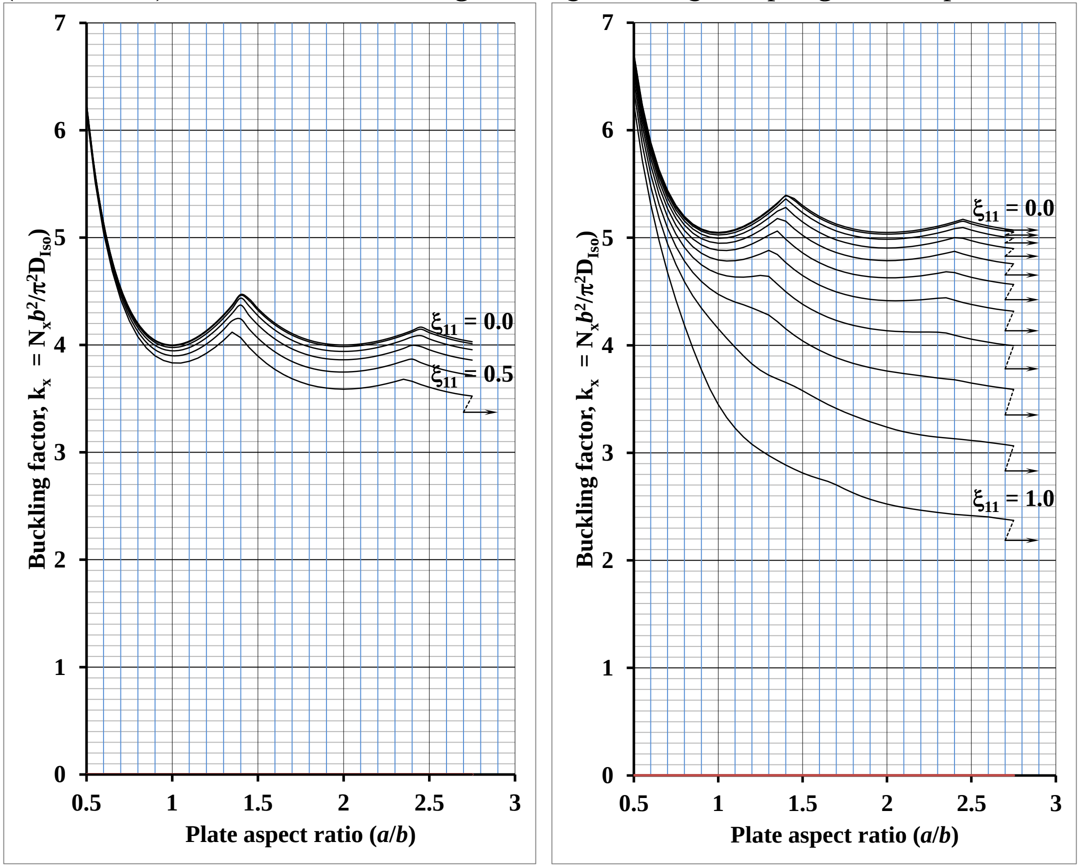


(c) $k_{x,\infty}$



$\xi_{11} = 0.5$

(Addendum) Influence of increasing *Bending-Twisting* Coupling on Compression Buckling Strength.



(a) (b)

Compression buckling factor curves for quasi-homogeneous ($D_{ij} = A_{ij}H^2/12$):

(a) quasi-isotropic laminates with (ξ_9, ξ_{10}) = (0,0);

(b) angle-ply laminates with (ξ_9, ξ_{10}) = (0,-1)

Asymptotes, representing $k_{x,\infty}$, reveal reductions of 16% for (a) and 57% for (b).

Table 7 – Results from the two ply termination algorithm applied to *Simple* ($\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$) laminates with even ply number groupings and transitioning to *Extension-Shearing Bending-Twisting* coupled ($\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F$) laminates between 12 and 14 plies.

(1)	(2a) $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$	(2b) $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F$	(3)	(4)	(5)	(6)	(7)	(8)	
16	260	5,927	63 (608) ↘	202 (-/72/130)			(8) **?**	8 (4/-/-)	$\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$
14	74	980	14 (132) ↘	50 (-/16/34)			(12) **4**	36 (-/12/24)	
12	22	203	4 (32) ↘	18 (-/2/16)	28 (32) ↘	106 (39/2/26)	(24) **24**	84 (35/2/12)	$\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F$
10	4	42	0 (6) ↘	-	6 (31) ↘	16 (6/-/4)	(20) **20**	38 (12/12/2)	
8	1	12	- (0)	-	- (10)	-	(10) **6**	-	

Column:

(1) Ply number grouping, *n*.

(2a) Column (2) results from Table 1.

(2b) Column (2) results from Table 5.

(3) Number of *n* ply laminates from Table 1 matching *n*-2 ply laminates of Table 5 after top-down termination scheme. Number of *n* ply laminates matching *n*+2 ply laminates are shown in parentheses.

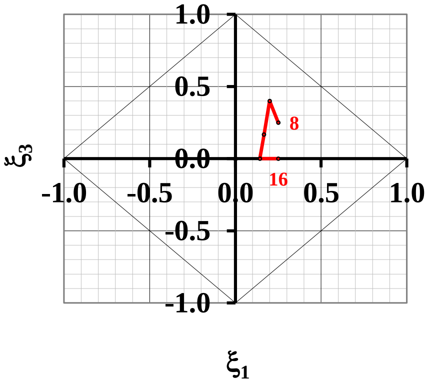
(4) Number of tapered solutions between adjacent ply number groupings (*n*) and orientation (○ or ●/+/–) of corresponding ply terminations.

(5) – (6) Repeat of (3) and (4) for Column (2b) results, beginning with result, in parentheses, from Column (3).

(7) Number of *n* ply laminates from (3) and (5) matching *n*+2 ply laminates after continuous bottom-up termination scheme. Number of *n* ply laminates matching *n*-2 ply laminates are shown in parentheses.

(8) Number of tapered solutions arising from (7) within each ply number grouping (*n*) and orientation (○ or ●/+/–) of corresponding ply terminations, for continuous tapering between 8-16 plies.

(Addendum) An tapered example, illustrated in the 3-dimensional space for the **extensional stiffness**:



Tapered laminate highlighted:

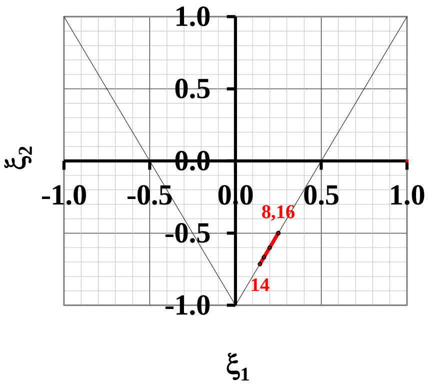
$(\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S)$: +/–/–/O/+/+/–/O/O/–/+/+/O/–/–/+

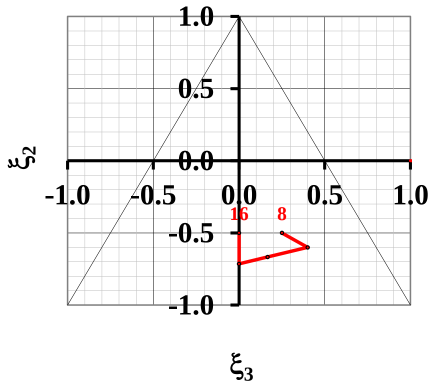
$(\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S)$: +/–/–/O/+/+/–/–/+/+/O/–/–/+

$(\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F)$: +/–/–/O/+/–/–/+/O/–/–/+

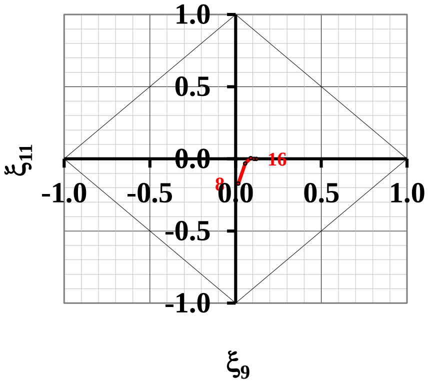
$(\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F)$: +/–/–/O/–/–/O/–/–/+

$(\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F)$: +/–/–/O/O/–/–/+





(Addendum) An example, illustrated in the 3-dimensional space for the **bending stiffness**:



Tapered laminate highlighted:

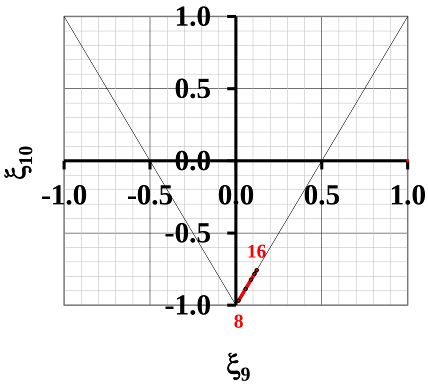
$(\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S)$: +/–/–/○/+/+/–/○/○/–/+/+/○/–/–/+

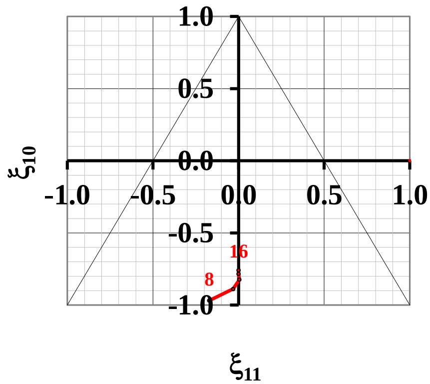
$(\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S)$: +/–/–/○/+/+/–/–/+/+/○/–/–/+

$(\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F)$: +/–/–/○/+/–/–/+/○/–/–/+

$(\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F)$: +/–/–/○/–/–/○/–/–/+

$(\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F)$: +/–/–/○/○/–/–/+





Concluding Remarks

- This article has investigated the extent of the design space for tapered warp-free laminates, with both uncoupled and mechanically coupled properties, when a two ply termination scheme is applied and ply contiguity (≤ 2) is accounted for. A number of key conclusions can be drawn from the results presented:

- Balanced angle-ply terminations are permissible in *Simple* and *Bending-Twisting* coupled laminate configurations with odd ply number groupings; only cross-ply termination are permissible in even ply number groupings. Such restrictions are not seen in *Extension-Shearing Bending-Twisting* coupled laminates.

- Lamination parameter point clouds can reveal strings of points representing the tapered designs found. These graphical representations of the design space offer insight into the potential for tailoring of stiffness properties through ply terminations.

- The transition between *Extension-Shearing Bending-Twisting* coupled laminates and *Simple* laminates has been demonstrated through a two ply termination scheme.

- Buckling strength comparisons, via a mapping of buckling factor contours onto the lamination parameter design space, can be used to assess the relative change in compression/shear buckling strength within the tapered laminate design

Acknowledgement

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